

Linear Algebra
Math 254
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Review for third exam
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Be able to use the following terminology

- eigenvalue, eigenvector (be able to define these also).
- basis for eigenspace.
- characteristic polynomial, characteristic equation.
- similar matrices.

Eigenvectors and Diagonalization

- Let A be an $n \times n$ matrix. You should be able to do the following.
 - Compute the characteristic polynomial of A .
 - Find the eigenvalues of A , when the characteristic polynomial is easily factored.
 - Find a basis for the eigenspace for each eigenvector.
 - Diagonalize A given n linearly independent eigenvectors.
 - When A is 2×2 , and has complex eigenvalues, find a rotation-scaling matrix that is similar to A . That is, if $a \pm bi$ are the eigenvalues, find P such that $P^{-1}AP = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$.
- Be able to use and understand the meaning of the main theorems.
 - A is diagonalizable if and only if it has n linearly independent eigenvectors.
 - If A has n distinct eigenvalues it is diagonalizable.
 - A matrix A is invertible if and only if 0 is *not* an eigenvalue of A .
 - Similar matrices have the same characteristic polynomial, and therefore the same eigenvalues with the same multiplicities.
- Be able to apply eigenvector analysis to a dynamical system.
 - Be able to classify a 2×2 matrix A :
Is the origin an attractor, a repeller, or a saddle point? Is A a rotation-contraction or a rotation-dilation? The latter cases occur when the eigenvalues are not real.
 - Be able to identify the long term behavior of a dynamical system, given the eigenvalues and eigenvectors.
 - Be able to write a transition matrix for a dynamical system given information about population changes.